Student's name   **Student's number**   **Teacher's name**

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PLC PRESBYTERIAN LADIES' COLLEGE SYDNEY 1888

2014
TRIAL
HIGHER SCHOOL CERTIFICATE
EXAMINATION

Mathematics

General Instructions

- Reading time - 5 minutes
- Working time - 3 hours
- Write using blue or black pen
  Black is preferred
- Board-approved calculators may be used
- A table of standard integrals is provided at the back of this paper
- All necessary working should be shown in every question

Total Marks – 100

Section I: Pages 3-6
10 marks

- Attempt questions 1-10, using the answer sheet on page 17.
- Allow about 15 minutes for this section

Section II: Pages 7-14
90 marks

- Attempt questions 11-16, using the lined paper provided.
- Allow about 2 hours 45 minutes for this section

Multiple Choice	11	12	13	14	15	16	Total
							%

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Section I

10 marks

Attempt Questions 1–10

Allow about 15 minutes for this section

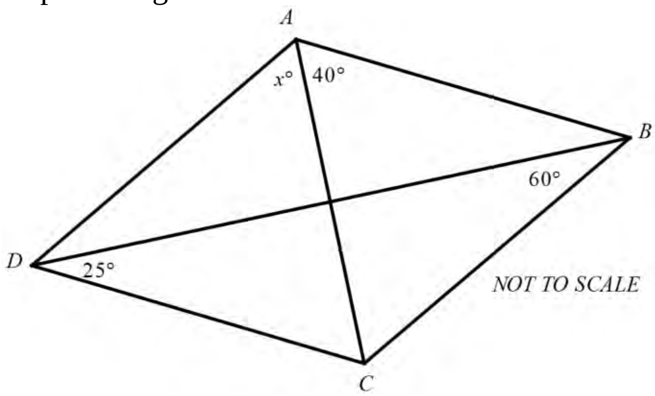
1. What is the equation of the straight line through $(4,5)$ and parallel to $2x-y+1=0$?

(A) $2x-y-13=0$
(B) $2x-y-3=0$
(C) $x+2y-10=0$
(D) $x+2y-7=0$

2. What are the nature of the roots of the quadratic equation $x^2-5x-6=0$?

(A) Real, rational and unequal
(B) Real, irrational and unequal
(C) Unreal, rational and unequal
(D) Unreal, irrational and unequal

3. *ABCD* is a parallelogram.



What is the value of *x?*

(A) 40
(B) 45
(C) 55
(D) 60

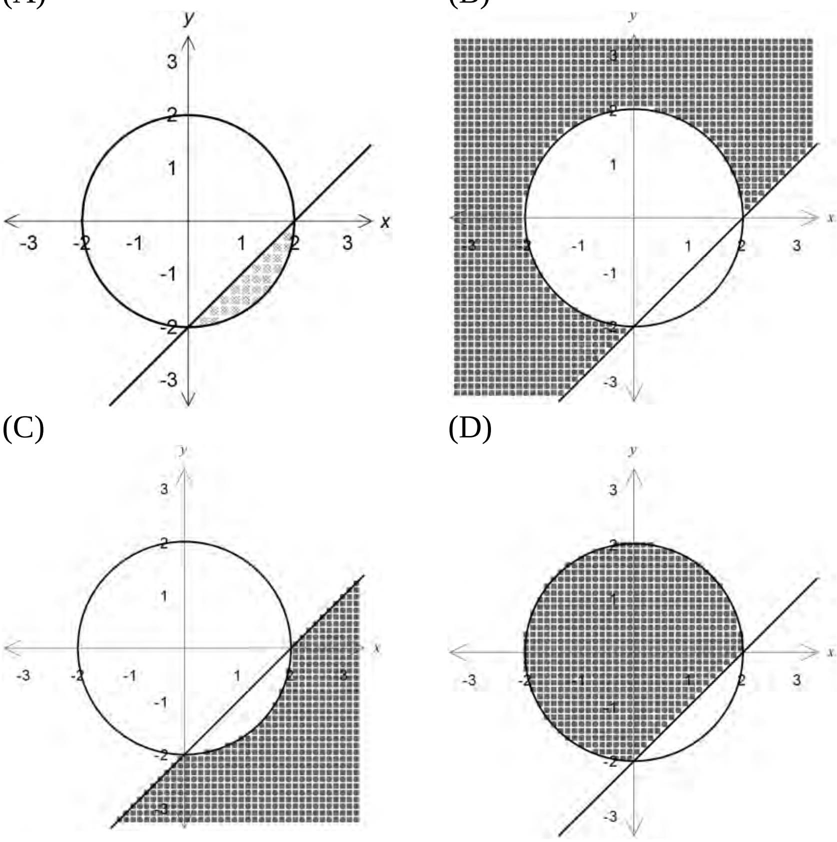
4. What is the equation of the directrix of the parabola $y^2 = -16(x-2)$?

(A) $x = -2$
(B) $x = 6$
(C) $y = -2$
(D) $y = 6$

5. Which of the following diagrams show where $x^2 + y^2 \geq 4$ and $y \leq x - 2$ hold simultaneously?

(A) (B)

(C) (D)



6. If $log_a 3 = -1.585$ and $log_a 5 = -2.322$, what is the value of $log_a\left(\frac{27a}{5}\right)$?

(A) -10.943
(B) -1.433
(C) 2.048a
(D) 6.143a

7. What is the equation of the curve that passes through $(4, 5)$ if the gradient function is $\sqrt{2x+1}$?

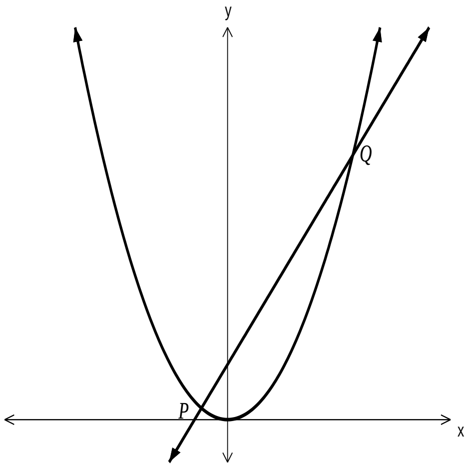
(A) $y=\frac{1}{3}(2x+1)^{\frac{3}{2}}-4$

(B) $x-3y+11=0$

(C) $3x-y-7=0$

(D) $y=\frac{2}{3}(2x+1)^{\frac{3}{2}}-13$

8. The x values of P and Q are the solutions to which quadratic equation?



(A) $x^2-3x-3=0$

(B) $x^2-3x+3=0$

(C) $x^2+3x-3=0$

(D) $x^2+3x+3=0$

9. If $\sin x=-\frac{1}{5}$ and $\pi \le x \le \frac{3\pi}{2}$, then $\cot x$ equals

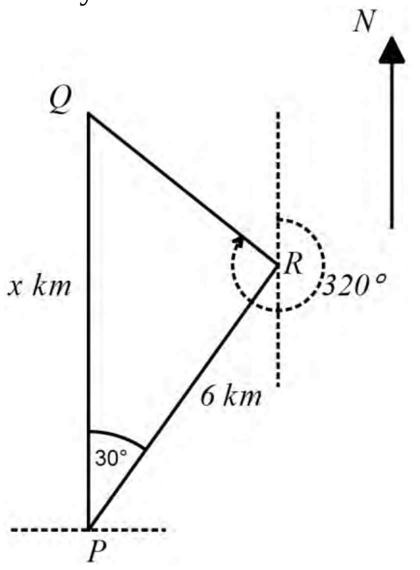
(A) $-\frac{1}{2\sqrt{6}}$

(B) $-2\sqrt{6}$

(C) $\frac{1}{2\sqrt{6}}$

(D) $2\sqrt{6}$

10. A ship leaves a port, P, and sails 6 *km* on a bearing of 030° to position R. It then heads on a bearing of 320° until it reaches a port, Q, which is directly north of P.



Which of the following will give the value for x?

(A) $\dfrac{x}{\sin 30^{\circ}} = \dfrac{6}{\sin 110^{\circ}}$

(B) $\dfrac{x}{\sin 40^{\circ}} = \dfrac{6}{\sin 110^{\circ}}$

(C) $\dfrac{x}{\sin 110^{\circ}} = \dfrac{6}{\sin 30^{\circ}}$

(D) $\dfrac{x}{\sin 110^{\circ}} = \dfrac{6}{\sin 40^{\circ}}$

Section II

90 marks

Attempt Questions 11–16

Allow about 2 hours 45 minutes for this section

Question 11 (15 marks) Use a SEPARATE writing booklet.

a) Solve for x: **2**

$$|3-2x| = 2x$$

b) Evaluate a and b if $\dfrac{2-\sqrt{5}}{2+\sqrt{5}} = a+\sqrt{b}$. **2**

c) Find $\lim_{x\to 0}\left[\dfrac{x^2-9x}{5x}\right]$ **1**

d) Find the domain of $y=\sqrt{3-2x-x^2}$. **2**

e) Differentiate the following with respect to x.

(i) $\dfrac{e^{x^3}}{5x}$ **2**

(ii) $\log_e\left(x^2+1\right)^{\frac{1}{2}}$ **2**

f) Find $\displaystyle\int \frac{3-x^2}{x}\,dx$ **2**

g) Show that $\displaystyle\int_0^{\log_e 2} \frac{2e^{2x}}{e^{2x}+1}\,dx = \log_e\left(\frac{5}{2}\right)$. **2**

End of Question 11

Question 12 (15 marks) Use a SEPARATE writing booklet.

a) Find the shortest distance between the lines $3x-5y=8$ and $3x-5y=-1$. **2**

b) If $f(x)=x^3-x+4$, find the value of $f\left(f(-1)\right)$. **2**

c) (i) Use Simpson's Rule with 3 function values to find an approximation to the area under the curve $y=\frac{1}{x}$ between $x=a$ and $x=3a$ where a is positive. **2**

(ii) Hence show that $log_e 3 \doteqdot \frac{10}{9}$. **1**

d) Sketch a continuous smooth curve for $x \geq 0$, where: **2**

$f(0)=1$,

$f'(x)<0$ and $f''(x)>0$ for $0<x<2$,

$f'(2)=0$,

$f(2)=-2$

$f'(x)>0$ and $f''(x)>0$ for $x>2$.

e) Prove $sec\,\theta - tan\,\theta - \frac{1}{sec\,\theta - tan\,\theta} = -2\,tan\,\theta$. **3**

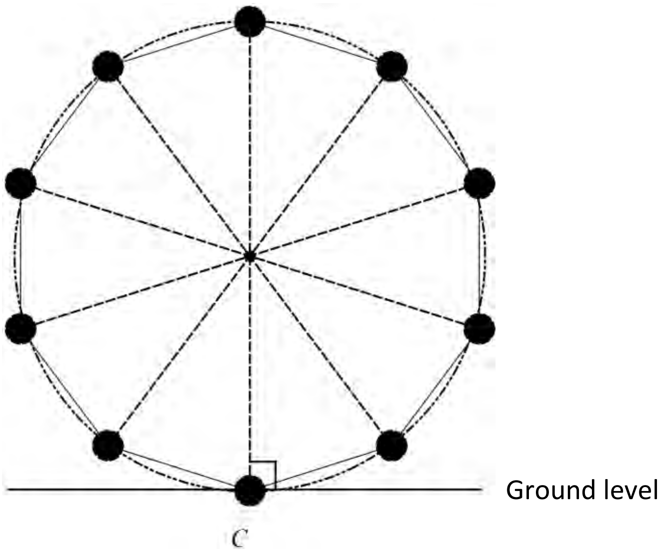
f) Find the equation of the locus of the point $P(x,y)$ such that the distance from *P* to the point *A*(2, 3) is twice the distance from *P* to the point *B*(-1, 4). Write your answer in simplest form. **3**

End of Question 12

Question 13 (15 marks) Use a SEPARATE writing booklet.

a) Find the value(s) of p such that $x^2+(p-1)x-(2p+1)>0$ for all values of x. **2**

b) A Ferris Wheel has a radius of 40 metres and 10 cages. A particular cage, C, starts at ground level and travels on a circular path.



(i) If the Ferris Wheel suddenly stops after cage C has moved 100 metres to a point D, through what angle has the wheel rotated? **1**

(ii) What is the area of the sector COD where O is the centre of the Ferris Wheel, C is the starting point of cage C and D is the point where cage C stopped? **1**

(iii) How far is cage C when it stops, in a straight line, from its starting point at ground level? Write your answer correct to 1 decimal place. **2**

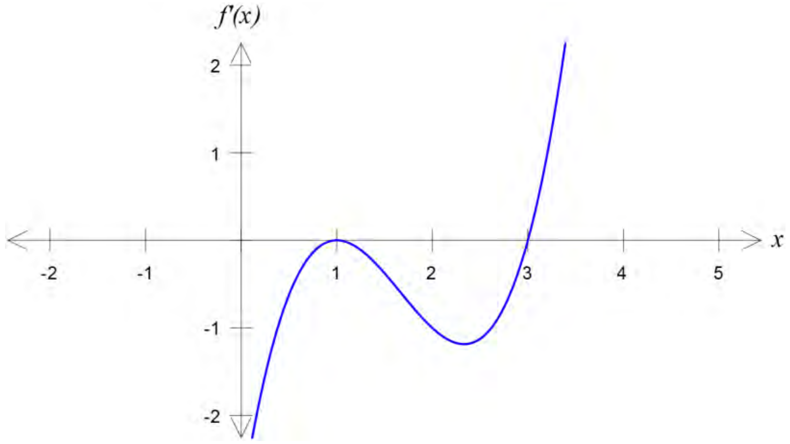
(iv) What is its height, to the nearest metre, above the ground now? **2**

c) The region between $y = sec\,x$ and the x-axis, bounded by $x=\frac{\pi}{6}$ and $x=\frac{\pi}{3}$ is rotated about the x-axis. Find the exact volume of the solid of revolution formed. **3**

Question 13 continues on page 10

d) If the roots of the equation $x^2 + ax + k = 0$ differ by $3a$, show that $k = -2a^2$ **2**

e) The graph of $y = f'(x)$ is shown below. **2**



In your answer booklet, draw $y = f(x)$, clearly showing any stationary points.

End of Question 13

Question 14 (15 marks) Use a SEPARATE writing booklet.

a) A particle moving in a straight line with its velocity, *v m/s* at time *t seconds* is given by $v = 3cos\left(2t - \frac{\pi}{2}\right)$.

(i) Show that the particle is initially at rest? **1**

(ii) What is the maximum speed of the particle? **1**

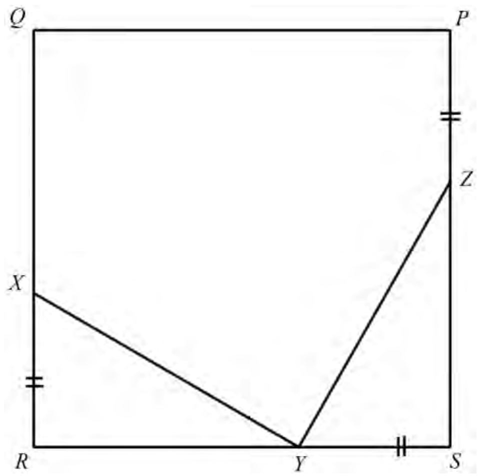
(iii) Find the first time the velocity of the particle reaches $\frac{3\sqrt{3}}{2}$ *m/s.* **2**

(iv) Find the acceleration of the particle at time *t*. **1**

(v) Sketch the acceleration of the particle as a function of time for $0 \le t \le \pi$. **2**

(vi) If the particle is initially at the origin, find its distance travelled in the first π seconds. **3**

b) In the diagram PQRS is a square.



(i) Prove that $\Delta XYR \equiv \Delta YZS$. **3**

(ii) Prove that $\angle XYZ = 90^o$. **2**

End of Question 14

Question 15 (15 marks) Use a SEPARATE writing booklet.

a) The population of feral pigs is growing at a rate proportional to the current population. The population of pigs, P, at time t years is given by, $P = P_0 e^{kt}$, where P_0 and k are constants.
In 2010, the feral pig population was first recorded.
The population in 2012 was 350 and in 2014 it was 410.

(i) Find the exact value of P_0 and k. **2**

(ii) What is the expected population of the pigs in 2020? **1**

(iii) In what year will the feral pig population reach 3000? **2**

b) ABC is an isosceles triangle in which $a = b = 1cm$. $\angle C$ is obtuse. The perpendicular from B to AC produced, meets AC in D so that $BD = \frac{1}{2}AD$. Let $\angle BCD = \theta$. Show that **2**

$$sin\theta = \frac{1 + cos\theta}{2}$$

c) For $y = 2x^2 e^x$

(i) Find the x and y intercepts, if any. **1**

(ii) What happens to the function as $x \to -\infty$? **1**

(iii) Show that stationary points exist at $(0, 0)$ and $\left(-2, \frac{8}{e^2}\right)$ **2**

(iv) Determine the nature of the stationary points. **2**

(v) It is known that 2 points of inflexion exist on this curve at $x = -2 \pm \sqrt{2}$. Sketch the curve. **2**

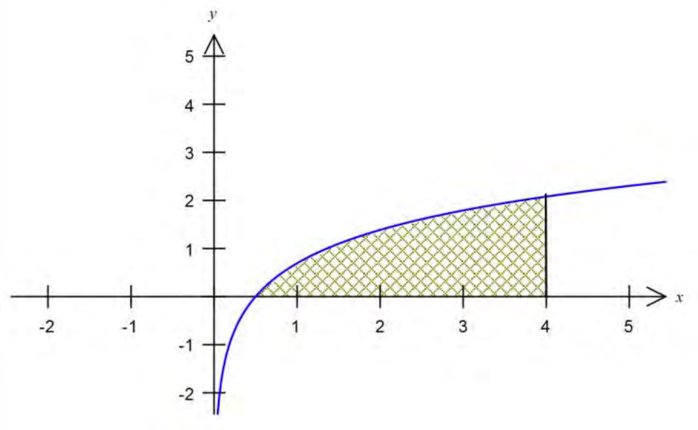
End of Question 15

Question 16 (15 marks) Use a SEPARATE writing booklet.

a) (i) Show that the equation of the normal to the parabola $x^2 = 16y$ at the point where $x = 4$ is $2x + y - 9 = 0$. **2**

(ii) A line parallel to this normal is a tangent to the parabola. Find its equation and the co-ordinates of the point of contact. **3**

b) Find the area under the curve $y = log_e 2x$, bounded by $x = 4$ and the x-axis. **3**

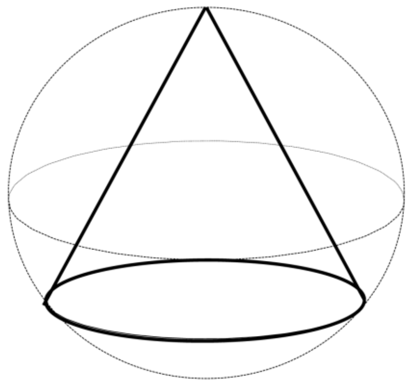


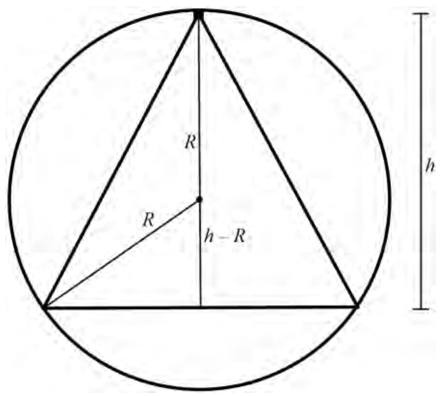
c) A tap is slowly turned on such that the volume flow rate of water, R, varies with time according to the relation $R = kt$, where k is a constant and $t > 0$. **2**
Calculate the total volume of water that flows from the tap in the first 10 seconds if $k = 1.3 m^3 / s^2$.

Question 16 continues on page 14

Question 16 continued

d) A right circular cone is inscribed in a sphere of radius R.





(i) Show that the volume of the cone can be found by **2**

$$V = \frac{1}{3}\pi\left(2Rh - h^2\right)h$$

(ii) Calculate the volume of the largest right circular cone inscribed in a sphere of radius R. Write your answer in terms of R. **3**

End of Paper

STANDARD INTEGRALS

$$\int x^n \, dx = \frac{1}{n+1} x^{n+1}, \quad n \neq -1; \; x \neq 0, \text{ if } n < 0$$

$$\int \frac{1}{x} \, dx = \ln x, \; x > 0$$

$$\int e^{ax} \, dx = \frac{1}{a} e^{ax}, \; a \neq 0$$

$$\int \cos ax \, dx = \frac{1}{a} \sin ax, \; a \neq 0$$

$$\int \sin ax \, dx = -\frac{1}{a} \cos ax, \; a \neq 0$$

$$\int \sec^2 ax \, dx = \frac{1}{a} \tan ax, \; a \neq 0$$

$$\int \sec ax \tan ax \, dx = \frac{1}{a} \sec ax, \; a \neq 0$$

$$\int \frac{1}{a^2 + x^2} \, dx = \frac{1}{a} \tan^{-1} \frac{x}{a}, \; a \neq 0$$

$$\int \frac{1}{\sqrt{a^2 - x^2}} \, dx = \sin^{-1} \frac{x}{a}, \; a > 0, \; -a < x < a$$

$$\int \frac{1}{\sqrt{x^2 - a^2}} \, dx = \ln\left(x + \sqrt{x^2 - a^2}\right), \; x > a > 0$$

$$\int \frac{1}{\sqrt{x^2 + a^2}} \, dx = \ln\left(x + \sqrt{x^2 + a^2}\right)$$

NOTE : $\ln x = \log_e x, \quad x > 0$

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Mathematics: Multiple Choice Answer Sheet

Student Number_________

Completely fill the response oval representing the most correct answer.

1.	**A** ⬭	**B** ⬭	**C** ⬭	**D** ⬭
2.	**A** ⬭	**B** ⬭	**C** ⬭	**D** ⬭
3.	**A** ⬭	**B** ⬭	**C** ⬭	**D** ⬭
4.	**A** ⬭	**B** ⬭	**C** ⬭	**D** ⬭
5.	**A** ⬭	**B** ⬭	**C** ⬭	**D** ⬭
6.	**A** ⬭	**B** ⬭	**C** ⬭	**D** ⬭
7.	**A** ⬭	**B** ⬭	**C** ⬭	**D** ⬭
8.	**A** ⬭	**B** ⬭	**C** ⬭	**D** ⬭
9.	**A** ⬭	**B** ⬭	**C** ⬭	**D** ⬭
10.	**A** ⬭	**B** ⬭	**C** ⬭	**D** ⬭

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Mathematics: Multiple Choice Answer Sheet

Student Number ANSWERS

Completely fill the response oval representing the most correct answer.

1.	A ○	B ●	C ○	D ○
2.	A ●	B ○	C ○	D ○
3.	A ○	B ○	C ●	D ○
4.	A ○	B ●	C ○	D ○
5.	A ○	B ○	C ●	D ○
6.	A ○	B ●	C ○	D ○
7.	A ●	B ○	C ○	D ○
8.	A ●	B ○	C ○	D ○
9.	A ○	B ○	C ○	D ●
10.	A ○	B ○	C ○	D ●

Academic Year	Yr 12	Calendar Year	2014
Course	2 Unit	Name of task/exam	Trials

Section I

1. $(4, 5)$ parallel to $2x - y + 1 = 0$

$y = 2x + 1$

$\therefore m_1 = 2$.

$m_2 = 2$ since $m_1 = m_2$ when parallel

$\therefore$ eqn

$y - y_1 = m(x - x_1)$

$y - 5 = 2(x - 4)$

$y - 5 = 2x - 8$

$2x - y - 3 = 0$

$\therefore B$

2. $x^2 - 5x - 6 = 0$

$\Delta = b^2 - 4ac$

$= (-5)^2 - 4(1)(-6)$

$= 25 + 24$

$= 49$

$\therefore \Delta > 0$ real roots
unequal roots

$\Delta = 49$ which is a perfect square
$\therefore$ rational roots

$\therefore A$

3. $\angle DCB = x + 40$ (opposite angles in parallelogram equal)

$\therefore 25 + 60 + x + 40 = 180$ (angle sum ΔDBC)

$x = 55$

$\therefore C$

4. $y^2 = -16(x - 2)$

of the form

$(y - k)^2 = -4a(x - h)$

$\therefore$ vertex $(2, 0)$

$4a = 16$

$a = 4$.

opens to left

$\therefore x = 6$

$\therefore B$

5. $x^2 + y^2 \geq 4$

means region outside circle

$\therefore B$ or C

$y \leq x - 2$

Test $(0, 0)$

$0 \nleq -2$

$(0, 0)$ not in region

$\therefore C$

6. $\log_a\left(\frac{27a}{5}\right) = \log_a 27a - \log_a 5$

$= \log_a 27 + \log_a a - \log_a 5$

$= \log_a 3^3 + 1 - \log_a 5$

$= 3\log_a 3 + 1 - \log_a 5$

$= -1.433$

$\therefore B$

Academic Year	Yr 12	Calendar Year	2014
Course	2U	Name of task/exam	Trials

Q7. $(4, 5)$ $\quad \frac{dy}{dx} = (2x+1)^{\frac{1}{2}}$

$$y = \int (2x+1)^{\frac{1}{2}} dx$$

$$y = \frac{(2x+1)^{\frac{3}{2}}}{\frac{3}{2}(2)} + c$$

$$y = \frac{(2x+1)^{\frac{3}{2}}}{3} + c$$

at $x = 4$ $\quad y = 5$

$$5 = \frac{9^{\frac{3}{2}}}{3} + c$$

$$5 = 9 + c$$

$$c = -4.$$

$$\therefore y = \frac{1}{3}(2x+1)^{\frac{3}{2}} - 4$$

$\therefore$ A

8. Parabola vertex $(0,0)$ must have eqn $y = x^2$

line with positive gradient and positive y-intercept must have eqn $y = 3x + 3$

$\therefore$ solutions are the points of intersection

$$x^2 = 3x + 3$$

$$x^2 - 3x - 3 = 0$$

(This is the quadratic eqn)

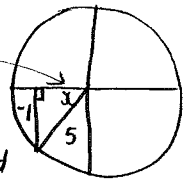
$\therefore$ A

9. $\sin x = -\frac{1}{5}$ $\quad \pi \leq x \leq \frac{3\pi}{2}$

3rd quad

$5^2 - (-1)^2 = a^2$

$a = \sqrt{24}$.



in 3rd quad

$a = -\sqrt{24}$.

$$\therefore \cot x = \frac{adj}{opp}$$

$$= \frac{-\sqrt{24}}{-1}$$

$$= \sqrt{24}$$

$$= 2\sqrt{6}$$

$\therefore$ D.

10. bearing $320°$ has $180° + \theta + \angle QRP$.

$\angle\theta$ is alternate to $\angle QPR$

$\therefore \angle\theta = 30°$

$$\therefore \angle QRP = 320 - 180 - 30$$

$$= 110°$$

$$\therefore \frac{a}{\sin A} = \frac{b}{\sin B}$$

$$\frac{x}{\sin 110} = \frac{6}{\sin 40}$$

$\therefore$ D

Academic Year	Yr 12	Calendar Year	2014
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Section II

Q11 a) $|3-2x| = 2x$

$3-2x = 2x$, $-3+2x = 2x$

$3 = 4x$, $-3 \neq 0$

$x = \frac{3}{4}$, no solution

Check $x = \frac{3}{4}$

$LHS = \left|3 - 2\left(\frac{3}{4}\right)\right|$

$= \left|3 - \frac{3}{2}\right|$

$= \frac{3}{2}$

$RHS = 2\left(\frac{3}{4}\right)$

$= \frac{3}{2}$

$\therefore LHS = RHS$

$\therefore x = \frac{3}{4}$ is solution

b) $\frac{2-\sqrt{5}}{2+\sqrt{5}} = a + \sqrt{b}$

$\frac{(2-\sqrt{5})}{(2+\sqrt{5})} \times \frac{(2-\sqrt{5})}{(2-\sqrt{5})} = \frac{4 - 4\sqrt{5} + 5}{4-5}$

$= \frac{9 - 4\sqrt{5}}{-1}$

$= 4\sqrt{5} - 9$

$\therefore 4\sqrt{5} - 9 = a + \sqrt{b}$

equating:

$a = -9$

$b = 16 \times 5 = 80$

c) $\lim_{x \to 0} \frac{x^2 - 9x}{5x}$

$= \lim_{x \to 0} \frac{\cancel{x}(x-9)}{5\cancel{x}}$

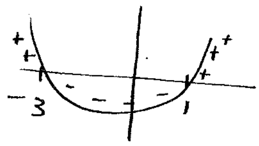
$= \frac{-9}{5}$

d) $y = \sqrt{3-2x-x^2}$

domain: $3 - 2x - x^2 \geq 0$

$x^2 + 2x - 3 \leq 0$

$(x-1)(x+3) \leq 0$



$-3 \leq x \leq 1$

e) i) $\frac{d}{dx} \frac{e^{x^3}}{5x}$

$= \frac{5x\left(3x^2 e^{x^3}\right) - e^{x^3}(5)}{(5x)^2}$

ii) $\frac{d}{dx} \ln\left(x^2+1\right)^{\frac{1}{2}}$

$= \frac{d}{dx} \frac{1}{2} \ln\left(x^2+1\right)$

$= \frac{1}{2} \frac{2x}{x^2+1}$

$= \frac{x}{x^2+1}$

OR $\frac{\frac{1}{2}\left(x^2+1\right)^{-\frac{1}{2}}(2x)}{\left(x^2+1\right)^{\frac{1}{2}}}$

Academic Year	Yr 12	Calendar Year	2014
Course	2 unit	Name of task/exam	Trials

f) $\int \frac{3 - x^2}{x} \, dx$

$= \int \left(\frac{3}{x} - x \right) dx$

$= \int \frac{3}{x} \, dx - \int x \, dx$

$= 3 \ln x - \frac{x^2}{2} + c$

g) RTS

$\int_0^{\ln 2} \frac{2e^{2x}}{e^{2x}+1} \, dx = \ln\left(\frac{5}{2}\right)$

LHS $= \int_0^{\ln 2} \frac{2e^{2x}}{e^{2x}+1} \, dx$

since $\int \frac{f'(x)}{f(x)} \, dx = \ln f(x)$

then

$= \left[\ln\left(e^{2x}+1\right) \right]_0^{\ln 2}$

$= \ln\left(e^{2\ln 2}+1\right) - \ln\left(e^0+1\right)$

$= \ln\left(e^{\ln 4}+1\right) - \ln(2)$

$= \ln(4+1) - \ln 2$

$= \ln 5 - \ln 2$

$= \ln\left(\frac{5}{2}\right)$

Q 12.

a) $3x - 5y = 8$

$3x - 5y = -1$

the shortest distance is the perpendicular distance.

A point on the line $3x-5y=8$ is $(1, -1)$

$\therefore$ perp. distance from this point to the other line is:

$d_\perp = \frac{|ax_1 + by_1 + c|}{\sqrt{a^2+b^2}}$

$= \frac{|3(1) - 5(-1) + 1|}{\sqrt{3^2+(-5)^2}}$

$= \left| \frac{3+5+1}{\sqrt{34}} \right|$

$= \frac{9}{\sqrt{34}}$

$= \frac{9\sqrt{34}}{34}$ units

b) $f(x) = x^3 - x + 4$

$f(f(-1)) = f\left((-1)^3 - (-1) + 4\right)$

$= f(-1+1+4)$

$= f(4)$

$= 4^3 - 4 + 4$

$= 64$

Academic Year	Yr 12	Calendar Year	2014
Course	2 unit	Name of task/exam	Trials

c) i) $A \doteqdot \frac{h}{3}\left[y_0 + y_n + 4y_1\right]$

$h = \frac{3a - a}{2} = a.$

$\therefore A = \frac{a}{3}\left[\frac{1}{a} + \frac{1}{3a} + 4 \times \frac{1}{2a}\right]$

$= \frac{a}{3}\left[\frac{1}{a} + \frac{1}{3a} + \frac{2}{a}\right]$

$= \frac{a}{3}\left[\frac{3+1+6}{3a}\right]$

$= \frac{a}{3}\left(\frac{10}{3a}\right)$

$= \frac{10}{9}$

ii) $\int_a^{3a} \frac{1}{x}\,dx = \left[\ln x\right]_a^{3a}$

$= \ln 3a - \ln a$

$= \ln\left(\frac{3a}{a}\right)$

$= \ln 3$

$\therefore \ln 3 \doteqdot \frac{10}{9}$

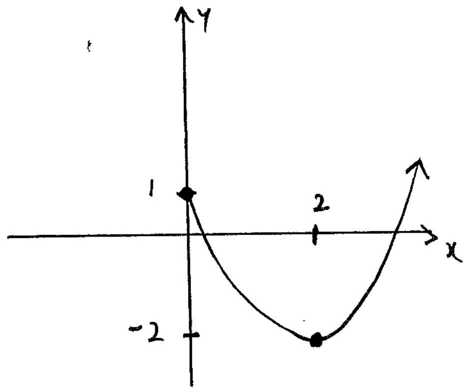
d) $f(0) = 1$, $f(2) = -2$, $f'(2) = 0$

$f'(x) < 0$ for $0 < x < 2$ decreasing

$f'(x) > 0$ for $x > 2$ increasing

$f''(x) > 0$ for $0 < x < 2$ concave up

$f''(x) > 0$ for $x > 2$ concave up



e) RTP

$\sec\theta - \tan\theta - \frac{1}{\sec\theta - \tan\theta} = -2\tan\theta$

LHS $= \frac{\sec\theta - \tan\theta}{1} - \frac{1}{\sec\theta - \tan\theta}$

$= \frac{(\sec\theta - \tan\theta)^2 - 1}{\sec\theta - \tan\theta}$

$= \frac{\sec^2\theta - 2\sec\theta\tan\theta + \tan^2\theta - 1}{\sec\theta - \tan\theta}$

$= \frac{\tan^2\theta + 1 - 2\sec\theta\tan\theta + \tan^2\theta - 1}{\sec\theta - \tan\theta}$ (using identity $\tan^2\theta + 1 = \sec^2\theta$)

$= \frac{2\tan\theta(\tan\theta - \sec\theta)}{(\sec\theta - \tan\theta)}$

$= -2\tan\theta$

$=$ RHS

f) $d_{PA} = 2 d_{PB}$

$$\sqrt{(x-2)^2+(y-3)^2} = 2\sqrt{(x+1)^2+(y-4)^2}$$

square both sides

$$(x-2)^2+(y-3)^2 = 4\left[(x+1)^2+(y-4)^2\right]$$

$$x^2-4x+4+y^2-6y+9 = 4\left[x^2+2x+1+y^2-8y+16\right]$$

$$\Rightarrow x^2-4x+y^2-6y+13 = 4x^2+8x+4+4y^2-32y+64$$

$$3x^2+12x+3y^2-26y+55=0$$

Q13

a) $x^2+(p-1)x-(2p+1)>0$

$\therefore$ positive def.

$a=1 \quad a>0$

$$\Delta = (p-1)^2-4(1)(-2p-1)$$

$$= p^2-2p+1-4(-2p-1)$$

$$= p^2-2p+1+8p+4$$

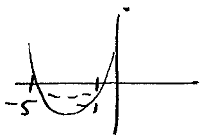
$$= p^2+6p+5$$

for pos. def $\Delta<0$

$\therefore p^2+6p+5<0$

$(p+1)(p+5)<0$

$-5<p<-1$



b) i) $l = r\theta$

$100 = 40\,\theta$

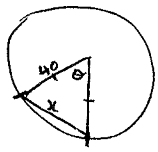
$$\theta = \frac{100}{40} = \frac{10}{4} = \frac{5}{2} \text{ radians}$$

ii) $A = \frac{1}{2}r^2\theta$

$$= \frac{1}{2}(40)^2\left(\frac{5}{2}\right)$$

$$= 2000 \text{ m}^2$$

iii)

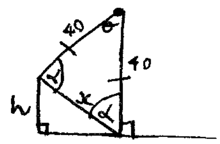


calculator in radians

$$x^2 = 40^2+40^2-2\times40\times40\cos\left(\frac{5}{2}\right)$$

$x = 75.9$ m (1 dp)

iv)



$$\alpha = \frac{\pi-\theta}{2} = \frac{\pi-\frac{5}{2}}{2} \doteqdot 0.32 \text{ rads}$$

$$\therefore \frac{\pi}{2} - 0.32\ldots = 1.25$$

$$\therefore \sin 1.25 = \frac{h}{x}$$

$$h = x\sin 1.25$$

$$= 75.9\times\sin 1.25$$

$$= 72.0 \text{ m.}$$

$= 72$ m (nrst metre).

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c) $V = \pi \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} (\sec x)^2 \, dx$

$= \pi \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \sec^2 x \, dx$

$= \pi \left[\tan x\right]_{\frac{\pi}{6}}^{\frac{\pi}{3}}$

$= \pi \left[\tan \frac{\pi}{3} - \tan \frac{\pi}{6}\right]$

$= \pi \left[\sqrt{3} - \frac{1}{\sqrt{3}}\right]$

$= \pi \left[\frac{3-1}{\sqrt{3}}\right]$

$= \frac{2\pi}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}}$

$= \frac{2\sqrt{3}\pi}{3}$ units^3

d) $x^2 + ax + k = 0$

if roots differ by $3a$

Let roots be α, β and $\alpha - \beta = 3a$

Sum of roots

$\alpha + \beta = -a$ ①

Product of roots

$\alpha\beta = k$ ②

also

$\alpha - \beta = 3a$ ③

Solve ① and ③

$2\alpha = 2a$

$\alpha = a$

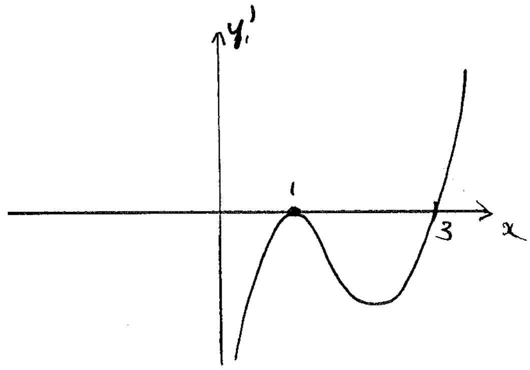
$\therefore \beta = -2a$

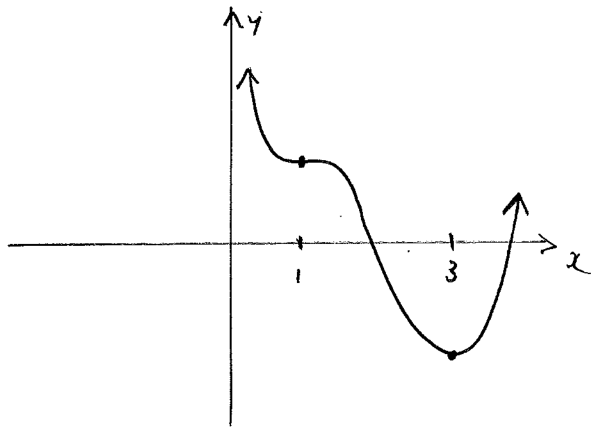
$\therefore \alpha\beta = k$ and

$\alpha\beta = a(-2a)$

$\therefore k = -2a^2$

e)





when $y' = 0$ (at $x = 1$)

x	0^-	0	0^+
y'	$-$	0	$-$

$-\!\!\searrow\!\!-$

when $y' = 0$ (at $x = 3$)

x	3^-	3	3^+
y'	$-$	0	$+$

$\therefore$ min

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Q14

a $v = 3\cos\left(2t - \frac{\pi}{2}\right)$

i if initally at rest then when $t = 0$, $v = 0$.

Show $t = 0$

$v = 3\cos\left(0 - \frac{\pi}{2}\right)$

$= 3\cos\left(-\frac{\pi}{2}\right)$

$= 3(0)$

$= 0$

$\therefore$ initially at rest.

ii max speed is when $v = 3$.

Note: $\cos\left(2t - \frac{\pi}{2}\right)$ is 1 as its maximum, & -1 as its minimum

iii $\frac{3\sqrt{3}}{2} = 3\cos\left(2t - \frac{\pi}{2}\right)$

$\frac{\sqrt{3}}{2} = \cos\left(2t - \frac{\pi}{2}\right)$

$-\frac{\pi}{6}, \frac{\pi}{6}, \frac{5\pi}{6}, \ldots = 2t - \frac{\pi}{2}$

$\frac{2\pi}{6}, \frac{4\pi}{6}, \frac{8\pi}{6}, \ldots = 2t$

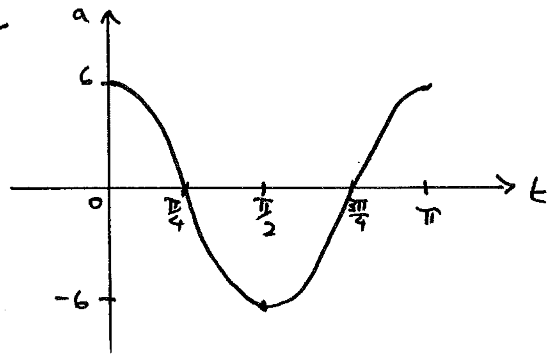
$t = \frac{\pi}{6}, \frac{2\pi}{6}, \ldots$

$\therefore$ first time $v = \frac{3\sqrt{3}}{2}$ is at $t = \frac{\pi}{6}$.

iv $a = -6\sin\left(2t - \frac{\pi}{2}\right)$

$\downarrow d$ s, c, −s, −c

v



vi $t = 0$ $x = 0$

$v = 3\cos\left(2t - \frac{\pi}{2}\right)$

$x = \frac{3\sin\left(2t - \frac{\pi}{2}\right)}{2} + c$

when $t = 0$ $x = 0$

$0 = \frac{3}{2}\sin\left(0 - \frac{\pi}{2}\right) + c$

$0 = -\frac{3}{2} + c$

$c = \frac{3}{2}$

$\therefore x = \frac{3}{2}\sin\left(2t - \frac{\pi}{2}\right) + \frac{3}{2}$.

between $t = 0$ and $t = \pi$ particle turns around at $t = \frac{\pi}{2}$.

$\therefore t = 0$ $x = 0$

$t = \frac{\pi}{2}$ $x = 3$

$t = \pi$ $x = 0$

$\therefore$ total distance travelled in first π seconds is 6 metres.

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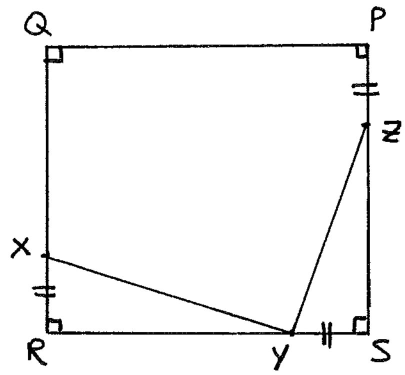
b)



i In $\triangle XYR$ and $\triangle YZS$

$\angle XRY = \angle YSZ$ (all angles in square $= 90°$)

$XR = YS$ (given)

$RY + YS = ZS + ZP$ (all sides in square are equal)

since $YS = PZ$ (given)

$RY = ZS$

$\therefore \triangle XYR \equiv \triangle YZS$ (SAS)

ii Let $\angle XYR = x$

$\therefore \angle YZS = x$ (in congruent triangles corresponding angles are equal)

$\therefore \angle ZYS = 90 - x$ (angle sum $\triangle ZYS$).

$\therefore \angle XYZ = 180 - x - (90 - x)$ (angle sum of straight line)

$\therefore \angle XYZ = 180 - x - 90 + x$

$= 90$

$\therefore \angle XYZ = 90°$.

Q15

a i $P = P_0 e^{kt}$

2010 records commences

t: 2012 $\Rightarrow$ $t = 2$ $P = 350$

t: 2014 $\Rightarrow$ $t = 4$ $P = 410$

$\therefore 350 = P_0 e^{k \times 2}$

$410 = P_0 e^{k \times 4}$

$\therefore 350 = P_0 e^{2k}$ ①

$410 = P_0 e^{4k}$ ②

$\frac{410}{350} = \frac{P_0 e^{4k}}{P_0 e^{2k}}$ ② ÷ ①

$\frac{41}{35} = e^{4k - 2k}$

$\frac{41}{35} = e^{2k}$

$\ln\left(\frac{41}{35}\right) = 2k$

$k = \frac{1}{2} \ln\left(\frac{41}{35}\right)$

$\therefore 350 = P_0 e^{\frac{1}{2}\ln\left(\frac{41}{35}\right) \times 2}$

$P_0 = \frac{350 \times 35}{41} = \frac{12250}{41}$

ii t: 2020 $\Rightarrow t = 10$

$P = 298.78\ldots e^{\frac{1}{2}\ln\left(\frac{41}{35}\right) \times 10}$

$P \doteqdot 659$

iii $3000 = 298.78 e^{\frac{1}{2}\ln\left(\frac{41}{35}\right) t}$

$\frac{3000}{298.78\ldots} = e^{\frac{1}{2}\ln\left(\frac{41}{35}\right) t}$

$\ln\left(\frac{3000}{298.78\ldots}\right) = \frac{1}{2}\ln\left(\frac{41}{35}\right) t$

$\therefore t = 29.2$ $\therefore 2040$

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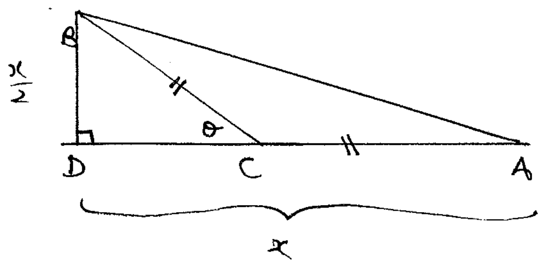
b



Let $BC = y$

$\therefore CA = y$.

$\therefore \sin\theta = \frac{BD}{BC}$

$\sin\theta = \frac{x/2}{y}$

$\therefore y = \frac{x}{2\sin\theta}$. $\therefore \frac{x}{y} = 2\sin\theta$

$\cos\theta = \frac{DC}{BC}$

$= \frac{x-y}{y}$

$\cos\theta = \frac{x}{y} - 1$

$\cos\theta = 2\sin\theta - 1$

$\therefore \sin\theta = \frac{\cos\theta + 1}{2}$

c $y = 2x^2e^x$

i for x-intercepts: set $y=0$

$\therefore 0 = 2x^2e^x$

$\therefore x=0$, $e^x \neq 0$

$\therefore x=0$

for y-intercept: set $x=0$

$\therefore y = 0e^0$

$y=0$

$\therefore (0,0)$ is x & y intercepts

ii as $x \to -\infty$

$y \to \frac{2x^2}{e^x} \to 0^+$

$y \to 0$

iii $\frac{dy}{dx} = uv' + vu'$ $u = 2x^2$, $u' = 4x$, $v = e^x$, $v' = e^x$

$= 2x^2e^x + e^x(4x)$

$= e^x(2x^2 + 4x)$

for stat pts $\frac{dy}{dx} = 0$

$e^x(2x^2+4x) = 0$

$\therefore e^x = 0$ (no sol^n) $2x^2+4x = 0$

$2x(x+2) = 0$

$\therefore x=0$, $x=-2$

$\therefore$ Stat pts at $(0,0)$, $\left(-2, \frac{8}{e^2}\right)$

iv

x	$-\frac{1}{2}$	0	$\frac{1}{2}$
$\frac{dy}{dx}$	-0.9	0	4.1

+ _/ +

$\therefore \min(0,0)$

x	-3	-2	$-\frac{1}{2}$
$\frac{dy}{dx}$	0.3	0	-0.9

+ /‾\ −

$\therefore \max\left(-2, \frac{8}{e^2}\right)$

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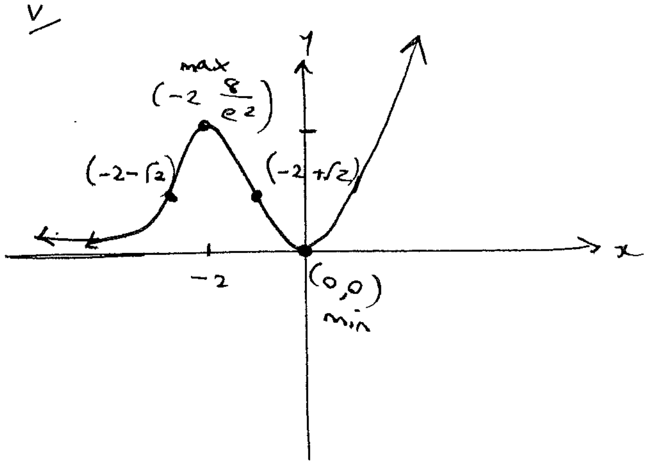
v



Q16

a i

$x^2 = 16y$

$y = \frac{x^2}{16}$

$\frac{dy}{dx} = \frac{2x}{16}$

at $x = 4$

$m_{tangent} = \frac{8}{16} = \frac{1}{2}$.

$\therefore m_{norm} = -2$

at $x = 4$ $y = \frac{4^2}{16} = 1$.

$\therefore (4, 1)$ is point

$\therefore$ eqn normal is

$y - 1 = -2(x - 4)$

$y - 1 = -2x + 8$

$2x + y - 9 = 0$.

is eqn of normal.

ii $y = \frac{x^2}{16}$

$\frac{dy}{dx} = \frac{2x}{16} = \frac{x}{8}$

if parallel to normal $m = -2$

$\frac{x}{8} = -2$

$\therefore x = -16$

at $x = -16$

$y = 16$

$\therefore (-16, 16)$ are the coordinates of the point of contact.

The equation is:

$y - 16 = -2(x + 16)$

$y - 16 = -2x - 32$

$2x + y + 16 = 0$

b $A = \int_a^b y \, dx$

$= \int_a^4 \ln 2x \, dx$

$= \text{rectangle} - \int_0^{\ln 8} x \, dy$

$= 4 \times \ln 8 - \int_0^{\ln 8} \frac{1}{2} e^y \, dy$

$= 4 \ln 2^3 - \left[\frac{1}{2} e^y\right]_0^{\ln 8}$

$= 12 \ln 2 - \left(\frac{1}{2} e^{\ln 8} - \frac{1}{2} e^0\right)$

$= 12 \ln 2 - \left(4 - \frac{1}{2}\right)$

$= 12 \ln 2 - \frac{7}{2}$

$y = \ln 2x$

$e^y = 2x$

$x = \frac{1}{2} e^y$

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c) $\frac{dV}{dt} = kt$

$V = \int kt\, dt$

$V = \frac{kt^2}{2} + c$

when $t = 0$ $V = 0$ $\therefore c = 0$

$\therefore V = \frac{1}{2}kt^2$

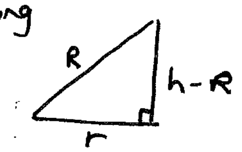
when $k = 1.3$, $t = 10$

$V = \frac{1}{2} \times 1.3 \times 10^2$

$V = 65\ m^3$

d) i) $V_{cone} = \frac{1}{3}\pi r^2 h$

radius of cone, r, is found using



$r^2 = R^2 - (h-R)^2$

$r^2 = R^2 - h^2 + 2hR - R^2$

$r^2 = 2hR - h^2$

$\therefore V_{cone} = \frac{1}{3}\pi(2hR - h^2)h$

ii) largest cone when sphere has radius R.

$\therefore R$ is constant

h is variable

$V = \frac{2}{3}\pi h^2 R - \frac{1}{3}\pi h^3$

$\frac{dV}{dh} = \frac{4}{3}\pi R h - \pi h^2$

for max/min $\frac{dV}{dh} = 0$

$\therefore h\left(\frac{4}{3}\pi R - \pi h\right) = 0$

$h = 0$, $\frac{4}{3}\pi R - \pi h = 0$

$\frac{4}{3}\pi R = \pi h$

$h = \frac{4R}{3}$

$\frac{d^2V}{dh^2} = \frac{4}{3}\pi R - 2\pi h$

when $h = 0$

$\frac{d^2V}{dh^2} = \frac{4}{3}\pi R$

> 0 ∪ $\therefore$ min

when $h = \frac{4R}{3}$

$\frac{d^2V}{dh^2} = \frac{4}{3}\pi R - 2\pi \frac{4R}{3}$

$= -\frac{4\pi R}{3}$

< 0 ∩ $\therefore$ max.

$\therefore$ Largest cone is when

$V = \frac{2}{3}\pi\left(\frac{4R}{3}\right)^2 R - \frac{1}{3}\pi\left(\frac{4R}{3}\right)^3$

$V = \frac{2}{3}\pi R\left(\frac{16R^2}{9}\right) - \frac{1}{3}\pi\left(\frac{64R^3}{27}\right)$

$V = \frac{32\pi R^3}{81}$ units^3